

## REMARKS/ARGUMENTS

In this rewritten application, Applicants have changed the title so as to properly place emphasis on the general linear controllers as disclosed in the old claim 13, which include PID controllers as special cases.

Applicants have rewritten the whole specification in a way required by the first and second paragraphs of 35 USC § 112, so as to overcome the technical rejections of the Office Action ("OA").

All old claims 1-15 are replaced by new claims 16-21, wherein new claims 16 and 18 replace old claim 13, new claims 17 and 19 replace old claim 14, new claim 20 replaces old claims 1 and 2, and new claim 21 replaces old claims 1 and 3.

### **Response to OA Rejections under the first paragraph of 35 USC 112**

In response to OA rejections in paragraph 2, page 2, Applicants have rewritten the specification in a way required by the first paragraph of 35 USC 112.

In the new specification formula for calculating the polynomials A and B in the open-loop transfer function  $A^{-1}B$  and formula for calculating the closed-loop transfer function Q are all explicitly and clearly given. From this given closed-loop transfer function Q, it is easy for any person with basic training in control theory to find the characteristic polynomial  $b(z)$  and then find the poles of Q by finding the roots of the characteristic equation  $b(z)=0$ . In the well-known commercial software product "Control System Toolbox" from The MathWorks Inc., the function "pole" can directly find the poles of a known transfer function such as the Q formulated in this invention. Another function "roots" in MATLAB from the MathWorks Inc

can also find all roots of a known polynomial such as  $b(z)$  in this invention. The function "minimax" or "fminimax" in the "Optimization Toolbox" from The MathWorks Inc. provides direct solution to the minimax or constrained minimax problem formulated in this invention. MATLAB and the related Toolboxes mentioned above are very well known among students and engineers of control engineering trained in a University. It is therefore unnecessary, for example, to present the mathematical theories and algorithms on how to find the roots of a polynomial and how to solve the minimax problem. These well-known mathematical tools can be considered known prior art and therefore do not need to be presented in this invention.

Therefore, following the rewritten specification, any person of ordinary skill in the art should now be able to make use of the invention without any difficulty.

For the above reasons, Applicants respectfully request further examination of Claims 16 and 18, which are rewritten forms of Claim 13, Claim 15 is deleted so as to focus this invention on discrete time controller design and tuning.

#### **Response to OA Rejections under the second paragraph of 35 USC 112**

Since Claims 4-12 and 15 are deleted, Applicants will only respond to OA rejections in paragraphs 7 and 8.

In response to OA rejection reasons in paragraph 7, page 3, the omitted element, i.e., the structure of the linear controller:

$$Du_k = Er_k - Cy_k$$

with detailed description is now added to both the rewritten specification and the new Claim

16 and, therefore, new Claims 17-21 since they are claims dependent on Claim 16.

In response to OA rejection reasons in paragraph 8, page 3, the specification is rewritten, Claim 13 is written as new Claims 16 and 18, and Claim 14 is rewritten as Claims 17 and 19.

Now both the rewritten specification and new Claims 16-19 clearly and positively described the structure of the linear controller. In the rewritten specification and the new Claims 16-19, every part of the whole closed-loop control system and the connection between the different parts of the whole system are now clearly and positively described.

Therefore, Applicants respectfully request further examination of the rewritten Claims 16-19, which are new forms of Claims 13 and 14.

#### **Response to OA Claim Rejections under the second paragraph of 35 USC 102 (b)**

For Claim 1, Applicants do not dispute that all MIMO (Multiple-Input Multiple-Output) PID controllers have Multiple inputs and Multiple outputs. This is true in Mori et al reference and Claim 1. But MIMO PID controllers differ from each other mainly in the different PID equations they use.

The PID equation used in Claim 1 is:

" $CO(k) = CO(k-1) + K1 * SP(k) * T + K1 * a(k,1) + K2 * a(k,2) + \dots + Kj * a(k,j)$ , where  $k$  is the discrete time,  $T$  is the sampling period,  $j$  is a positive integer,  $K1, K2, \dots, Kj$  are  $m$  by  $n$  PID parameters,  $a(k,1) = [-PV(k)] * T$ , and  $a(k,j) = [a(k,j-1) - a(k-1,j-1)]/T$  for  $j > \text{or} = 2$ "

This PID equation has  $j$  terms and is in discrete time format, where  $j$  is an integer.

Applicants have read every paragraph of the Mori et al reference and haven't found any evidence to show that Mori et al also uses this PID equation or similar PID equation. On the contrary, Mori et al column 2 lines 51-55 "digital PID parameters  $K_c$ ,  $T_i$  and  $T_d$ " shows their PID is the traditional PID controller with three terms, while PID equation in Claim 1 has  $j$  terms and  $j \geq 2$ . Mori et al reference column 2 lines 29-32, lines 36-40, column 3-4 lines 62-74 and figure 3 all show that the PID controller in Mori et al reference is a PID controller with multiple inputs and multiple outputs. It does not show that the Mori et al reference uses the same PID equation as in Claim 1.

Therefore, Applicants respectfully request withdrawal of the rejection of Claim 1 under 35 USC 102 (b), and respectfully request further examination of new Claim 20, which is a rewritten form of the combination of Claims 1 and 2.

For Claim 2, Applicants submit that the Mori et al reference column 5 lines 66-67 "... PID control parameters are obtained from the S transfer function ..." and column 8 lines 9-13 "... PID parameters ... are tuned in accordance with the identified transfer function  $G_p(s)$ " and column 8 lines 60-62 "The S-transfer function of the control system between the set-point signal  $R(s)$  and the process output  $Y(s)$  is matched with the reference model  $M(s)$ " all say that Mori et al use the continuous-time transfer function, i.e., the Laplace S-transfer function, to find the tuning parameters, while Claim 2 uses the discrete time transfer function, i.e., the Z-transfer function, to find the tuning parameters. The two are totally different. Further, Applicants have read every paragraph of the Mori et al reference and haven't found any evidence to show that the Mori et al reference uses the tuning method as described in Claim 2. So the OA erred in saying that (see page 5 lines 15-18) "As per claim2, the Mori et al reference discloses the  $m$  by  $n$  PID parameters  $K_1$ ,  $K_2$ , ..., and  $K_j$  are obtained by using an optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV." Hence, the OA erred in saying (see page 4, lines 12-13) "Claims 1, 2, 4, 5, 7, 8, 10, 11 and 13 are

rejected under 35 USC 102(b) as being anticipated by USPN 4,563,734 Mori et al.”

Therefore, Applicants respectfully request withdrawal of the rejection of Claim 2 under 35 USC 102 (b), and respectfully request further examination of new Claim 20, which is a rewritten form of the combination of Claims 1 and 2.

For Claim 13, since the rejection of Claim 2 is incorporated in rejection of Claim 13, for the same reasons as stated above, Applicants respectfully request withdrawal of the rejection of Claim 13 under 35 USC 102 (b), and respectfully request further examination of new Claims 16 and 18, which are rewritten forms of Claim 13.

#### **Response to OA Rejections under 35 USC 103 (a)**

For Claims 1 and 2, Applicants submit the following two facts

**Fact 1:** The following findings by the OA show that none of the references provided by the OA disclosed or suggested, whether expressly or impliedly, the PID equation and the tuning method in this invention, or anything close to the present invention:

For Claim 1:

- (1) (Page 9 of the OA action) The Katebi et al reference page 1457 Introduction Section: “...variables of a given sub-process.... Local controller set points.... System variables at desired values ... Boilers have multiple numbers of inputs and outputs” mainly states that boilers have multiple numbers of inputs and outputs. No connection with the tuning method or PID equation in this invention at all.
- (2) (Page 9 of the OA action) The Katebi et al reference page 1459: “Optimal Control Based Methods: Wang and Wu [9] have proposed a multi-objective optimization method to calculate the parameters of the PID controller... an optimal procedure for

selecting the PID parameters of a multiloop controller. This method is based on centralized LQ problem." The LQ method has no connection with the tuning method in this invention at all.

- (3) (Page 9 of the OA action) The Katebi et al reference page 1460: "Non Parametric Methods: "... controlled variables are measurable, the classes of input disturbances and reference inputs are known, and the system is controllable by a diagonal PID controller." This is just a trivial description of use of multivariable PID controllers. No connection with the tuning method or PID equation in this invention at all.
- (4) (Page 9 of the OA action) The Katebi et al reference page 1460: "Davison Method: "... tuning matrices for a multivariable controller ... closed-loop system ..." The Davison Tuning Method described here (see the tuning equation (13) in this reference) is totally different from that of this invention.
- (5) (Page 9 of the OA action) The Katebi et al reference page 1460: "Generalized Ziegler-Nichols Method: "... proposed an extension of SISO ... to MIMO system." The Ziegler-Nichols tuning method is also totally different from that in this invention.
- (6) (Page 9 of the OA action) OA agrees that "The M.R. Katebi et al reference does not expressly disclose the PID control equation of the present invention" (see OA action page 9 lines 15-16)

For Claim 2:

- (7) (Page 11 of the OA action) The Toru Yamamoto et al reference page 126: "The tuning of the control parameters in PID control laws ... a self-tuning PID control algorithm based on the generalized minimum variance control scheme ..." This method is trying to minimize the performance index as in a generalized minimum variance control scheme, which is also totally different from the present invention.
- (8) (Page 11 of the OA action) The Toru Yamamoto et al reference page 126: "PID tuning... closed-loop input-output relationship..." This is a trivial description of the authors' derivation procedure, which has no connection with the present invention.

- (9) (Page 11 of the OA action) OA agrees that "the M.R. Katebi et al reference does not expressly disclose the m by n PID parameters  $K_1, K_2, \dots, K_j$  are obtained by using an optimization algorithm which minimizes the largest modulus of all poles of the discrete time closed loop transfer function from said SP to said PV." (see OA action page 11, lines 8-11)
- (10) Neither the Katebi et al reference nor the Yamamoto et al reference nor any other references provided by the OA suggested or implied the tuning method as disclosed in Claim 2, or would motivate any person skilled in the art to develop the tuning method as in Claim 2.

**Fact 2:** Ever since early 1940s when PID controllers gained wide acceptance in industry, a huge number of researchers and engineers have been involved in the research and development of PID controllers and their tuning. However, until today no one has published this very successful PID tuning method as presented in the present invention (Claim 2) or mentioned the structure of the PID in Claim 1. This fact clearly shows that neither Claim 1 nor Claim 2 is obvious to a person of ordinary skill in the art because otherwise this very successful tuning method would have been discovered for a long time. Therefore, OA **erred** in saying (see page 8 lines 3-4) "Claims 1, 4, 7, and 10 are rejected under 35 USC 103 (a) as being unpatentable over M. R. Katebi et al..." and (see page 11, lines 1-7) "Claims 2, 5, 8, 11 and 13 are rejected under 35 USC 103 (a) as being unpatentable over M.R. Katebi et al... in view of Toru Yamamoto et al..."

From the above **Fact 1** and **Fact 2**, Applicants respectfully request the withdrawal of rejections of Claim 1 and especially Claim 2 under 35 USC 103 (a), and respectfully request further examination of new Claim 20, which is a combination of Claim 1 and Claim 2.

Since the OA rejection of Claim 13 is incorporated with Claim 2 (see page 12, lines 14-16), the same reasons as stated above apply to Claim 13. Therefore, Applicants respectfully

request the withdrawal of rejections of Claim 13 under 35 USC 103 (a), and respectfully request further examination of new Claims 16 and 18, which are the rewritten forms of Claim 13.

For Claim 3, please note the following facts:

- (1) The OA agrees that the modified teachings of Katebi et al and Yamamoto et al do not expressly disclose the tuning method as described in Claim 3 (see OA action page 13, lines 10-15).
- (2) Indeed neither the Katebi et al reference nor the Yamamoto et al reference suggests in any way the tuning method in Claim 3 or will motivate any person skilled in the art to come up with the tuning method in Claim 3, because the tuning methods in these references are totally different from Claim 3 or any other Claims of this invention (for example, the objective functions are totally different.).
- (3) The tuning method proposed in the G. Celentano et al. reference "... minimize the greatest closed loop time constant imposing constraints not only on controller parameters but also on overshoot and control signal amplitude ..." is totally different from Claim 3 since the objective function in the Celentano et al reference is totally different from that in Claim 3. Neither does the Celentano et al reference suggest Claim 3 in any way.
- (4) The tuning method in Claim 3 has been very successful. However, although a huge number of researchers and engineers have been studying the PID controller tuning problem ever since the early 1940s, no one else proposed the tuning method as described in Claim 3, even until today. Therefore, Claim 3 is not obvious to a person of ordinary in the art.

Therefore, OA erred in saying (see page 13, lines 1-9) "Claims 3, 6, 9, 14 and 15 are rejected under 35 USC 103 (a) as being unpatentable over M.R.Katebi et al... in view of



Toru Yamamoto et al...."

Therefore, Applicants respectively request OA withdrawal of rejection of Claim 3 under 35 USC 103 (a), and respectfully request further examination of Claim 21, which is the rewritten form of Claim 3.

Since the OA rejection of Claim 14 is incorporated with rejection of Claim 3 (see page 15, lines 1-3), the same reasons as stated above apply to Claim 14. Therefore, Applicants respectfully request the withdrawal of rejections of Claim 14 under 35 USC 103 (a), and respectfully request further examination of Claims 17 and 19, which are rewritten forms of Claim 14.

Applicants have also read the references provided by OA, as listed on pages 16-17. Applicants find that

- (1) None of these references discloses anything that is similar to any claim in this invention, whether expressly or impliedly,
- (2) USPN 6,510,353 B1 to Gudaz et al discloses a tuning method by use of a robustness map in a simulation program. USPN 5,680,304 to Wang et al has nothing to do with PID controller tuning. All other references listed on pages 16-17 deal with PID tuning problem in the S-transfer function domain, which are different from the present invention since this invention solves the linear controller tuning problem in the discrete time domain.

## Conclusion

For all of the above reasons, Applicants respectfully submit that the specification and claims are now in proper form and that the claims all define patentably over the prior art. Therefore

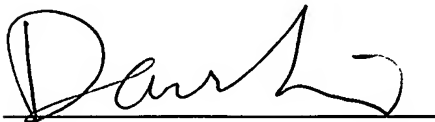
they submit that this application is now in condition for allowance.

Applicants sincerely thank the helpful criticisms of the Examiner. Those criticisms, especially those given in paragraphs 2-8 of the Office Action, directly motivated the Applicants to rewrite the whole application.

#### **Conditional Request For Constructive Assistance**

Applicants have rewritten the specification and claims of this application so that they are proper, definite, and define novel structure which is also unobvious. If for any reason this application is not believed to be in full condition for allowance, Applicants respectfully request the constructive assistance and suggestions of the Examiner pursuant to M.P.E. § 2173.02 and § 707.07(j) in order that the undersigned can place this application in allowable condition as soon as possible and without the need for further proceedings.

Very respectfully,



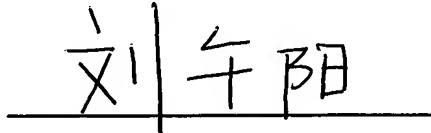
Danyang Liu

#108, 10615-111 Street

Edmonton, Alberta T5H 3G1

Canada

Telephone: (780) 424-5074



Wuyang Liu

P.O. Box 112

316, San Jing Lu

Nanchang, P. R. China

Mark-up copy

**Version Showing Changes Made**

Please note: Because we are using the Cancellation-And-Substitution method to amend the application, you will not see any markings.

**In the Specification**

Please replace the whole old specification with the following completely rewritten new specification:

**SPECIFICATION****TITLE OF THE INVENTION**

Methods of Designing Optimal Linear Controllers

**TECHNICAL FIELD OF THE INVENTION**

This invention relates to the design of optimal multivariable linear controllers and PID controllers.

**BACKGROUND OF THE INVNTION**

A PID (proportional-integral-derivative) controller is a device, usually implemented in a computer, that is used to control a process variable (e.g., temperature, pressure, etc) of an industrial process. For a multiple-input and multiple-output (MIMO) process, the process variable  $y(t)$  and controller output  $u(t)$  are  $n$ -dimensional vector variables of the time  $t$ . The controller receives the sample values of  $y(t)$  and calculates its output  $u(t)$  according to a PID equation. The controller sends its output  $u(t)$  to the process so that the error  $e(t)=r(t)-y(t)$  approaches zero as time  $t$  increases, where  $r(t)$  is the set point (also known as the reference signal or the command signal, etc).

There are many types of PID controllers, depending on the use of different types of PID equations. In discrete time form, all these PID controllers can be viewed as special cases of the general linear controller with the linear control equation

$$Du_k = Er_k - Cy_k$$

where  $y_k = y(t_k)$  is the process variable at time  $t_k = t_0 + kT_s$ ,  $t_0$  is the initial time,  $T_s > 0$  is the constant sampling period,  $k = 0, 1, 2, \dots$  is a non-negative integer called discrete time variable,  $u_k = u(t_k)$  is the controller output at time  $t_k$ , and  $u_k$  can be subject to upper limit and/or lower limit constraints placed on one or more of its components,  $r_k = r(t_k)$  is the set point at time  $t_k$ ,  $D$ ,  $E$  and  $C$  are  $n$  by  $n$  matrix polynomials in the unit backward shifting operator  $z^{-1}$  such that for any discrete time signal  $x_k$ ,  $z^{-1}x_k = x_{k-1}$ , and one or more of the said  $D$ ,  $E$  and  $C$  contain tuning parameters that are to be determined,

For example, if  $D = (1 - z^{-1})/T_s \cdot I$ , where  $I$  is the identity matrix of order  $n$ , and  $E = C = K_i + K_p D + K_d D^2$ , where  $K_i$ ,  $K_p$  and  $K_d$  are user specified constant coefficient matrices called the integral gain, the proportional gain and the derivative gain, then the linear controller becomes the following "type A" PID controller

$$Du_k = K_i e_k + K_p D e_k + K_d D^2 e_k$$

where  $e_k = r_k - y_k$  is the error. The three gains  $K_i$ ,  $K_p$  and  $K_d$  are the tuning parameters.

If the aforesaid  $E$  is changed to  $K_i + K_p D$  or  $K_i$ , then the linear controller becomes the following "type B" and "type C" PID controllers, respectively:

$$Du_k = K_i e_k + K_p D e_k - K_d D^2 y_k$$

$$Du_k = K_i e_k - K_p D y_k - K_d D^2 y_k$$

Once its structure is selected, the performance of a linear controller or a PID controller depends mainly on the choice of its tuning parameters.

How to properly choose the tuning parameters for a PID controller is a problem that has attracted a lot of studies ever since PID controllers became widely used in industry in the early 1940s. The Ziegler-Nichols tuning methods developed by Ziegler and Nichols in 1942 and 1943 are still widely used in industry. Other model based methods choose the tuning parameters by minimizing some well-known control performance index such as the integral absolute errors (IAE), the integral squared errors (ISE), etc. However, practice shows that all these methods often lead to oscillatory control results.

#### DETAILED DESCRIPTION OF THE INVENTION

Suppose the open-loop transfer function of the process from  $u_k$  to  $y_k$  is  $A^{-1}B$ , where  $A$  and  $B$  are known  $n$  by  $n$  matrix polynomials in the unit backward shifting operator  $z^{-1}$ :

$$\begin{aligned} A &= I - A_1 z^{-1} - A_2 z^{-2} - \dots - A_p z^{-p} \\ B &= B_1 z^{-1} + B_2 z^{-2} + \dots + B_q z^{-q} \end{aligned}$$

where  $I$  is the identity matrix of order  $n$ ,  $p$  and  $q$  are two known positive numbers, and  $A_1, A_2, \dots, A_p, B_1, B_2, \dots, B_q$  are constant coefficient matrices of order  $n$ .  $A$  and  $B$  can be obtained by use of the commercially available software product "System Identification Toolbox" from The MathWorks Inc. Below is another way for finding  $A$  and  $B$ .

Select a positive integer  $v$  that is not less than  $n(p+q)$  and let  $k$  be not less than  $v + \max\{p, q\}$ , where  $\max\{p, q\}$  denotes the larger one of the  $p$  and  $q$ . Let  $h_k$  be a vector defined by

$$h_k^T = [y_{k-1}^T \quad y_{k-2}^T \quad \dots \quad y_{k-p}^T \quad u_{k-1}^T \quad u_{k-2}^T \quad \dots \quad u_{k-q}^T \quad 1]$$

where for any matrix  $X$ ,  $X^T$  denotes the transpose of  $X$ , and let

$$Y = [y_k \quad y_{k-1} \quad \dots \quad y_{k-v}]^T$$

$$H = [h_k \quad h_{k-1} \quad \dots \quad h_{k-v}]^T$$

$$W = \text{diag}(1, w, w^2, \dots, w^v)$$

where  $w$  is a positive real number not larger than one, and  $W$  is a diagonal matrix with 1,  $w$ ,  $w^2$ , ..., and  $w^v$  on its diagonal. The number  $k$  should be large enough and the controller output signal  $u_k$  should be chosen so as to ensure that the matrix  $H$  is of full column rank. Then the matrices  $A_1, A_2, \dots, A_p, B_1, B_2, \dots$ , and  $B_q$ , and the vector  $d$  can be obtained from

$$[A_1 \quad A_2 \quad \dots \quad A_p \quad B_1 \quad B_2 \quad \dots \quad B_q \quad d] = ((WH)^+(WY))^T$$

where for any matrix  $X$ ,  $X^+$  denotes the Moore-Penrose pseudo-inverse of  $X$ , which is a standard function under the name "pinv" in the software product MATLAB from The MathWorks Inc. (MATLAB is a registered trademark of The MathWorks Inc.). Thus the two polynomials  $A$  and  $B$  and a vector  $d$  are obtained. When  $A$ ,  $B$ , and  $d$  are so obtained, the process can be approximately represented by the following linear model:

$$Ay_k = Bu_k + d$$

And the open-loop transfer function of the process is then  $A^{-1}B$ . Please note that this invention does not deal with the problem of how to get the polynomials  $A$  and  $B$ . From now on it is always assumed that, one way or another, the two polynomials  $A$  and  $B$  are known.

If the process is controlled by the following linear controller as described in the previous

section:

$$Du_k = Er_k - Cy_k$$

where D, E, and C are n by n polynomials in the backward shifting operator  $z^{-1}$ , and one or more of D, E and C contain tuning parameters that are to be specified, then the closed-loop transfer function from the set point  $r_k$  to the process variable  $y_k$  is

$$Q = (A + BD^{-1}C)^{-1}BD^{-1}E$$

From Q the characteristic polynomial in z, denoted by  $b(z)$ , can be found. All roots of the characteristic equation  $b(z)=0$ , denoted by  $z_1, z_2, z_3, \dots, z_j$ , where j is a positive number, form all the poles of Q.

Let  $\max \{|z_1|, |z_2|, \dots, |z_j|\}$  denote the maximum of all the absolute values  $|z_1|, |z_2|, \dots$ , and  $|z_j|$  of the poles of Q. Then the best choice of the tuning parameters is such that the  $\max \{|z_1|, |z_2|, \dots, |z_j|\}$  is minimized, i.e., the best tuning parameters form a solution to the following minimax problem:

$$\min \max \{|z_1|, |z_2|, \dots, |z_j|\}$$

In situations where one or more of the tuning parameters must be within user specified regions, the above minimax problem becomes a constrained minimax problem.

Many commercially available software products can directly be used to find the poles of a transfer function and the roots of a polynomial, and to solve the minimax or constrained minimax problem as formulated above, such as (1) the function "pole" in the "Control System Toolbox", which can find the poles directly from the transfer function Q, (2) the

function "roots" in MATALAB, which can directly find the roots of the characteristic equation  $b(z)=0$ , and (3) the functions "minimax" and "fminimax" in the "Optimization Toolbox", which directly provides solution to the constrained or unconstrained minimax problem formulated above, all of which are easy to use and commercially available from The MathWorks Inc.

### In the Claims

Please cancel claims 1 to 15 and substitute new claims 16 to 21 as follows:

16 A method for determining the optimal tuning parameters in a linear controller, wherein

- a) the said linear controller is a device that receives an n-dimensional process variable  $y(t)$  from a process and sends an n-dimensional controller output signal  $u(t)$  to the said process, where  $t$  is the time variable and  $n$  is a positive integer,
- b) the said linear controller uses the following type of linear difference equation to calculate the controller output  $u_k$

$$Du_k = Er_k - Cy_k$$

where  $y_k = y(t_k)$  is the process variable at time  $t_k = t_0 + kT_s$ ,  $t_0$  is the initial time,  $T_s > 0$  is the constant sampling period,  $k$  is a non-negative integer called discrete time variable,  $u_k = u(t_k)$  is the controller output at time  $t_k$ , and  $u_k$  can be subject to lower limit and/or upper limit constraints placed on one or more of its components,  $r_k = r(t_k)$  is the set point at time  $t_k$ ,  $D$ ,  $E$  and  $C$  are  $n$  by  $n$  matrix polynomials in the unit backward shifting operator  $z^{-1}$  such that for any discrete time signal  $x_k$ ,  $z^{-1}x_k = x_{k-1}$ , and one or more of the said  $D$ ,  $E$  and  $C$  contain tuning parameters that are to be determined,



- c) the said discrete time open-loop transfer function of the said process from the said controller output  $u_k$  to the said process variable  $y_k$  is  $A^{-1}B$ , where  $A$  and  $B$  are known  $n$  by  $n$  matrix polynomials in the unit backward shifting operator  $z^{-1}$ , and
- d) the said method finds the optimal values for the said tuning parameters by minimizing the maximum of absolute values of all poles of the discrete time closed-loop transfer function  $(A+BD^{-1}C)^{-1}BD^{-1}E$  from the said set point  $r_k$  to the said process variable  $y_k$ .
- 17 A method as in Claim 16, wherein the said minimization of the maximum of absolute values of all poles of the closed loop transfer function  $(A+BD^{-1}C)^{-1}BD^{-1}E$  from the said set point  $r_k$  to the said process variable  $y_k$  is subject to constraints placed on the said tuning parameters.
- 18 A method as in Claim 16, wherein  $D=(1-z^{-1})/T_s \cdot I$ , where  $I$  is the identity matrix of order  $n$ ,  $E=K_1$ ,  $C=K_1$  or  $C=K_1+K_2D$  or  $C=K_1+K_2D+K_3D^2$  or  $C=K_1+K_2D+K_3D^2+K_4D^3$  or  $C=K_1+K_2D+K_3D^2+K_4D^3+\dots+K_mD^{m-1}$ , where  $m$  is a positive integer, and the coefficients  $K_1, K_2, \dots, K_m$  are  $n$  by  $n$  constant matrices and are the said tuning parameters.
- 19 A method as in Claim 17, wherein  $D=(1-z^{-1})/T_s \cdot I$ , where  $I$  is the identity matrix of order  $n$ ,  $E=K_1$ ,  $C=K_1$  or  $C=K_1+K_2D$  or  $C=K_1+K_2D+K_3D^2$  or  $C=K_1+K_2D+K_3D^2+K_4D^3$  or  $C=K_1+K_2D+K_3D^2+K_4D^3+\dots+K_mD^{m-1}$ , where  $m$  is a positive integer, and the coefficients  $K_1, K_2, \dots, K_m$  are  $n$  by  $n$  constant matrices and are the said tuning parameters.
- 20 A linear controller as in Claim 16 with its structure and tuning parameter determined by Claim 18 and  $m>3$ .

- 21 A linear controller as in Claim 16 with its structure and tuning parameter determined by Claim 19 and  $m > 3$ .

**In Abstract**

Please cancel the old abstract and substitute the new abstract as follows:

-- Methods of designing optimal discrete time linear controllers are disclosed. The optimal values of the tuning parameters in a linear controller can be found by minimizing the maximum of absolute values of all poles of the discrete time closed-loop transfer function from the set point to the process variable. This is a minimax problem and can be solved without any difficulty. A constrained optimization can be used to solve this minimax problem in cases where one or more of the tuning parameters are subject to lower limit and/or upper limit constraints. The PID (proportional-integral-derivative) controller tuning problem is thus solved since PID controllers can be considered as special cases of the general linear controller.--